## Gradient

A road gradient of $25 \%$ can be written as $\frac{25}{100}=\frac{1}{4}$ or 'one in four'. This means the road rises by 1 m for every 4 m of horizontal run.

1. Complete this table. In the final column, rank the gradients 1 to 8 (where 1 is the steepest).

2. In the US, the 'grade' of a road is its slope written as a percentage. A warning sign must be posted if a section of road has a grade of at least $8 \%$ and is more than 750 feet long.
a) A road rises 63 feet over a horizontal distance of 840 feet. Should a warning sign be posted? Explain your answer.
b) The grade of a section of road that stretches over a horizontal distance of 1,000 feet is $9 \%$. How many feet does the road rise over that distance?
3. In the UK, the law states that wheelchair ramps should have a gradient no steeper than 1:12.
a) A builder is constructing a wheelchair ramp up to a front door that is 20 inches off the ground. Since material is expensive, the builder wants to use the least amount of material, but also meet the legal maximum gradient requirement. How far will the ramp extend horizontally?
b) You measure another ramp and discover that it has a horizontal run of 160 inches and is 10 inches high. Does this ramp meet the legal requirements?
4. What is the angle of elevation of a road with gradient $100 \%$ ? Explain how you know.

When you finish: Use stacks of books to demonstrate the various gradients listed in Question 1

## Teacher Notes

See my blog post 'All about gradient' for related teaching ideas.
http://mathsteachingtips.blogspot.com/2014/06/all-about-gradient.html
Knowledge of trigonometry is not required for this worksheet, but if pupils have studied trigonometry in right-angled triangles then you could ask them to calculate the angle of elevation of each road in question 1.

## Sources

Some questions on this worksheet were taken from:
http://ohsmathdept.weebly.com/uploads/1/8/0/0/18007225/8.4.pdf
http://tidewaterteam.wm.edu/files/2012/06/slope.pdf

## Answers

1. Point out to students that the lowest n in the ratio 1 : n gives the steepest gradient

|  | Gradient as a <br> percentage | Gradient as a <br> simplified fraction | Gradient as a ratio <br> $\mathbf{1}: \mathbf{n}$ | Steepness ranking <br> (steepest = 1) |
| :---: | :---: | :---: | :---: | :---: |
| A | $10 \%$ | $\frac{1}{10}$ | $1: 10$ | 6 |
| B | $30 \%$ | $\frac{3}{10}$ | $1: 3.3$ | 2 |
| C | $20 \%$ | $\frac{1}{5}$ | $1: 5$ | 3 |
| D | $14 \%$ | $\frac{7}{50}$ | $1: 7.14$ | 4 |
| E | $8 \%$ | $\frac{2}{25}$ | $1: 12.5$ | 7 |
| F | $12.5 \%$ | $\frac{1}{8}$ | $1: 8$ | 5 |
| G | $5 \%$ | $\frac{1}{20}$ | $1: 20$ | 8 |
| H | $100 \%$ | $1: 1$ | 1 |  |

2. In the US, the 'grade' of a road is its slope written as a percentage. A warning sign must be posted if a section of road has a grade of at least $8 \%$ and is more than 750 feet long.
a) A road rises 63 feet over a horizontal distance of 840 feet. Should a warning sign be posted? Explain your answer.
Grade $=($ rise $\div$ run $) \times 100=(63 \div 840) \times 100=7.5 \%$.
The road is more than 750 feet but the grade is less than $8 \%$ so no warning sign is necessary.
b) The grade of a section of road that stretches over a horizontal distance of 1,000 feet is $9 \%$. How many feet does the road rise over that distance?

$$
\frac{\text { rise }}{1000}=0.09 \therefore \text { rise }=1000 \times 0.09=90 \text { feet }
$$

3. In the UK, the law states that wheelchair ramps should have a gradient no steeper than 1:12.
a) A builder is constructing a wheelchair ramp up to a front door that is 20 inches off the ground. Since material is expensive, the builder wants to use the least amount of material, but also meet the legal maximum gradient requirement. How far will the ramp extend horizontally?

He wants the ramp to be as steep as possible, therefore the gradient should be 1:12. $1: 12$ is equivalent to $20: 240$. The rise is 20 inches so he needs a run of $\mathbf{2 4 0}$ inches.
b) You measure another ramp and discover that it has a horizontal run of 160 inches and is 10 inches high. Does this ramp meet the legal requirements?

Yes, it has a $10: 160=1: 16$ ratio which is less steep than the required 1:12 ratio
4. What is the angle of elevation of a road with gradient $100 \%$ ? Explain how you know.

45 degrees. A right-angled triangle with base 1 and height 1 is isosceles (see sketch).


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